A Practical Guide for Consistency in Valuation:

Cash Flows, Terminal Value and Cost of Capital

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Abstract

Practitioners and teachers very easily break some consistency rules when doing or teaching valuation of assets. In this short and simple note we present a practical guide to call the attention upon the most frequent broken consistency rules. They have to do firstly with the consistency in the matching of the cash flows, this is, the free cash flow (FCF), the cash flow to debt (CFD), the cash flow to equity (CFE) and the tax savings or tax shield (TS). Secondly, they have to do with the proper expression for the cost of unlevered equity with finite cash flows and perpetuities. Thirdly, they have to do with the consistency between the terminal value and growth for the FCF and the terminal value and growth for the CFE, when there is a jump in the CFE due to the adjustment of debt to comply with the leverage at perpetuity. And finally, the proper determination of the cost of capital either departing from the cost of unlevered equity (Ku) or the cost of levered equity (Ke). In the Appendixes we show some algebraic derivations and an example.

Key Words

Cash flows, free cash flow, cash flow to equity, valuation, levered value, levered equity value, terminal value, cost of levered equity, cost of unlevered equity.

JEL Classification

M21, M40, M46, M41, G12, G31, J33

Introduction

Practitioners and teachers very easily break some consistency rules when doing or teaching valuation of assets. In this short and simple note we present a practical guide to call the attention upon the most frequent broken consistency rules. They have to do firstly with the consistency in the matching of the cash flows, this is, the free cash flow (FCF), the cash flow to debt (CFD), the cash flow to equity (CFE), the capital cash flow (CCF) and the tax savings or tax shield (TS). Secondly, they have to do with the proper expression for the cost of levered equity, Ke and different formulations for the weighted average cost of capital, WACC, with finite cash flows and perpetuities. Thirdly, they have to do with the consistency between the terminal value and growth for the FCF and the terminal value and growth for the CFE when there is a jump in the CFE due to the adjustment of debt to comply with the leverage at perpetuity. And finally, the proper determination of the cost of capital either departing from the cost of unlevered equity (Ku) or the cost of levered equity (Ke).

This note is organized as follows: In Section One, we present the consistency of cash flows and values according to the Modigliani and Miller propositions. In Section Two we define consistency in terms of cash flows and values. In Section Three we show the different expressions for Ke, traditional WACC and adjusted WACC for perpetuities and finite cash flows. In Section Four we mention the relationship between the terminal value, TV for the FCF and the TV for the CFE. In Section Five we show how to proceed for the estimation of Ku and Ke. In Section Six we conclude. In the Appendixes we derive the basic algebraic expressions and we illustrate these ideas with a simple example.

Section One

The Modigliani–Miller (M&M) Proposal

The basic idea is that the firm value does not depend on how the stakeholders finance it. This is the stockholders (equity) and creditors (liabilities to banks, bondholders, etc.) They proposed that with perfect market conditions, (perfect and complete information, no taxes, etc.) the capital structure does not affect the value of the firm because the equity holder can borrow and lend and thus determine the optimal amount of leverage. The capital structure of the firm is the combination of debt and equity in it.

That is, V^L the value of the levered firm is equal to V^{UL} the value of the unlevered firm.

$$V^L = V^{UL} \qquad (1)$$

And in turn, the value of the levered firm is equal to V^{Equity} the value of the equity plus V^{Debt} the value of the debt.

$$V^L = V^{Equity} + V^{Debt} \qquad (2)$$

This situation happens when no taxes exist. To maintain the equality of the unlevered and levered firms, the return to the equity holder (levered) must change with the amount of leverage (assuming that the cost of debt is constant)

When corporate taxes exist (and no personal taxes), the situation posited by M&M is different. They proposed that when taxes exist the total value of the firm does change. This occurs because no matter how well managed is the firm if it pays taxes, there exist what economists call an externality. When the firm deducts any expense, the government pays a subsidy for the expense. The value of the subsidy is the tax savings. (See Vélez–Pareja and Tham, 2003)

Hence the value of the firm is increased by the present value of the tax savings or tax shield.

$$V^{L} = V^{UL} + V^{TS} \tag{3}$$

Then, combining equations (2) and (3), we have

$$V^{UL} + V^{TS} = V^{Debt} + V^{Equity} \tag{4}$$

Although we have combined equations (3) and (4), it has to be said that V^{Equity} in equation (2) is different from V^{Equity} in equation (4). What we do is to combine the concept of the equality between levered value, V^{L}, and the values of debt and equity.

Each of the values in equation (4) has an associated cash flow as follows:

Table 1. Correspondence between values and cash flows

Market value	Cash flow
Assets	Free Cash Flow FCF
Debt	Cash Flow to Debt CFD
Equity	Cash Flow to Equity CFE
Tax savings	Cash Flow of TS

According to (4) then the cash flows will be related as

$$FCF + TS = CFD + CFE \tag{5}$$

and

$$FCF = CFD + CFE - TS \tag{6}$$

This is derived from the following reasoning:

Be X the FCF (identical to the operating profit) for an unlevered firm in perpetuity and no corporate taxes. Then the value of the firm is

$$V = \frac{X}{Ku} \tag{7a}$$

Where V is the value of the firm, Ku is the cost of the unlevered equity and X is the FCF.

If the firm is levered, then the value of debt is

$$D = \frac{CFD}{Kd} \tag{7b}$$

Where D is the market value of debt, CFD is the cash flow to debt and Kd is the cost of debt.

The CFD is Kd = Kd×D and the CFE is

$$CFE = NI = X - Kd \times D \tag{7c}$$

Where NI is net income.

$$\text{Hence, } FCF = X = Kd \times D + CFE = CFD + CFE \tag{7d}$$

For an unlevered firm with no corporate taxes then

$$FCF = CFD + CFE \tag{7e}$$

If we have taxes, then CFE is

$$CFE = NI = X - Kd \times D - T \times (X - Kd \times D) = X + T \times Kd \times D - Kd \times D - T \times X$$

$$= X \times (1-T) + T \times Kd \times D - Kd \times D \tag{7f}$$

Where T is the corporate tax rate.

And this is

$$CFE = X \times (1-T) + T\ Kd \times D - Kd \times D = X \times (1-T) + TS - CFD$$

$$= FCF \text{ (after taxes)} + TS - CFD \tag{7g}$$

Or, with taxes taken into account,

$$FCF + TS = CFD + CFE \tag{7h}$$

And (7h) is identical to (6)[1].

The FCF is the amount available for distribution to the capital owners (debt and equity) after an adjustment for tax savings. The CFE is what the stockholders receive as

[1] Although we have presented the value conservation first and the cash flow conservation as derived from it, the conservation of cash flow is the prior relationship. The value conservation follows from the conservation of cash flow, not the other way around.

dividends or equity repurchase and what they invest in the firm. The CFD is what the firm receives or pays to the debt holders. The TS is the subsidy the firm receives from the government for paying interest. The sum of what the owners of the capital is named as Capital Cash Flow (CCF) and is equal to the sum of the CFD and the CFE.

How do we discount these cash flows? In this table we indicate which discount rate to use for each cash flow.

Table 2. Correspondence between cash flows and discount rates

Cash flow	Discount rate
CFD	Cost of debt, Kd
CFE	Cost of levered equity, Ke
FCF	WACC
TS	The appropriate discount rate for TS, ψ
CCF	The appropriate discount rate for the CCF

Section Two

Consistency

Our purpose is to provide the correct procedures and expressions for the different inputs in valuing a cash flow and to guarantee the consistency between the cash flows and the market values. For consistency we understand the following result:

Total levered value = PV(FCF) = PV(CFE) + Debt = PV(CCF) (8a)

In other words,

Market equity value = Total Levered value – Debt = PV(FCF) − Debt

= PV(CFE) = PV(CCF) − Debt (8b)

Total levered value = APV = PV(FCF at Ku) + PV(TS at ψ) (8c)

These relationships have to hold for any year.

Section Three

The Proper Cost of Capital: Which Discount Rate for TS we can Use

In this section we list the proper definitions for Ke and WACC for perpetuities and finite cash flows taking into account which discount rate we use for TS, ψ. We will consider only two values for ψ = Ku and Kd.

In the following tables we list the different cases for Ke, traditional WACC and adjusted WACC. We consider simple perpetuities (no growth), finite cash flows (the most common situation) and ψ equal to Ku and to Kd. Each of these sets of formulas is presented to be applied to the FCF, to the CFE and to the CCF.

Applied to the FCF and to the CFE:

The general expression for Ke is

$$Ke_i = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} - (Ku_i - \psi_i)\frac{V^{TS}_{i-1}}{E^L_{i-1}} \qquad \text{(9a)}^2$$

Where Ke is the levered cost of equity, Ku is the unlevered cost of equity, Kd is the cost of debt, D is the market value of debt, E is the market value of equity, ψ is the discount rate for the TS and V^{TS} is the present value of the TS at ψ.

From this expression we can have the following: the formulation when ψ_i is Kd or Ku. If ψ_i is Ku_i the third term in the right hand side (RHS) of equation 9a vanishes, and the expression for Ke is

$$Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} - (Ku_i - Kd_i)\frac{V^{TS}_{i-1}}{E^L_{i-1}} \qquad \text{(9b)}$$

If ψ_i is Kd_i then

[2] See appendix A for derivation.

$$Ke = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} - (Ku_i - Kd_i)\frac{V^{TS}_{i-1}}{E^L_{i-1}} \quad (9c)$$

Now we have to consider two cases: perpetuities (simple) and finite cash flows.

When we have perpetuities, we have to remind that the present value of the perpetuity for TS is TD because TS is T×Kd×D (T is the corporate tax rate) and the present value when discounted at Kd is simply T×D. If cash flows are not perpetuities, they are finite and we have to use expression 9b.

When we simplify 9c for perpetuities and ψ_i is Kd_i we have

$$Ke = Ku_i + (Ku_i - Kd_i)\left[\frac{D_{i-1}}{E^L_{i-1}} - \frac{V^{TS}_{i-1}}{E^L_{i-1}}\right] \quad (9d)$$

But V^{TS} is

$$PV(TS) = \frac{T \times Kd \times D}{Kd} = TD \quad (9e)$$

And

$$Ke = Ku_i + (Ku_i - Kd_i)\left[\frac{D_{i-1}}{E^L_{i-1}} - \frac{T \times D_{i-1}}{E^L_{i-1}}\right] \quad (9f)$$

When we simplifying we obtain

$$Ke = Ku_i + (Ku_i - Kd_i)(1 - T)\frac{D_{i-1}}{E^L_{i-1}} \quad (9g)$$

This is a well known formula for Ke, but it applies only to perpetuities. When we do not have perpetuities we have to use 9d.

Table 3. Return to levered equity Ke according to the Discount rate for TS

	$\psi_i = Ku_i$	$\psi_i = Kd_i$
Perpetuity	$Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}}$	$Ku_i + (Ku_i - Kd_i)(1-T)\frac{D_{i-1}}{E^L_{i-1}}$
Finite	$Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}}$	$Ku_i + (Ku_i - Kd_i)\left(\frac{D_{i-1}}{E^L_{i-1}} - \frac{V^{TS}_{i-1}}{E^L_{i-1}}\right)$

When we examine the weighted average cost of capital, WACC, we can handle the problem in a similar way. The only prevention is to include the proper Ke formulation in its calculation. Let us call this WACC, WACC for the FCF, $WACC^{FCF}$.

Table 4. Traditional $WACC^{FCF}$ formula for the FCF according to the Discount rate for TS

	$\psi_i = Ku_i$	$\psi_i = d_i$
Perpetuity and Finite	$Kd_i(1-T)\frac{D_{i-1}}{V^L_{i-1}} + \frac{Ke_i E_{i-1}}{V^L_{i-1}}$	$Kd_i(1-T)\frac{D_{i-1}}{V^L_{i-1}} + \frac{Ke_i E_{i-1}}{V^L_{i-1}}$

In this traditional formulation V is the market value of the firm; other variables have been defined above.

We have to warn the reader about the correctness of the traditional WACC. The previous table shows the typical and best know formulation for WACC, but it has to be said that this formulation is valid only for a precise and special case: when there is enough earnings before interest and taxes (EBIT) to earn the TS, that the TS are earned in full and that taxes are paid the same year as accrued. To cover deviations from this special case we can use a more general formulation for WACC:

$$WACC_{adjusted} = Ku_i - \frac{TS_i}{V^L_{i-1}} - [(Ku_i - \psi_i)\frac{V^{TS}_{i-1}}{V^L_{i-1}} \qquad (9h)^3$$

If $\psi_i = Ku_i$ the third term in the RHS of equation 9b vanishes. If $\psi_i = Kd_i$ reminding that the present value of the perpetuity for TS is TD and simplifying we obtain

[3] See appendix A for derivation.

$$WACC_{adjusted} = Ku_i - \frac{TS_i}{V^L_{i-1}} - [(Ku_i - Kd_i)\frac{TD}{V^L_{i-1}}$$

When the discount rate for TS is Kd we have to use 9h. The derivation of the formulas in the next table can be read in the Appendix.

Table 5. Adjusted $WACC^{FCF}$ formula for the FCF according to the Discount rate for TS

	$\psi_i = Ku_i$	$\psi_i = Kd_i$
Simple perpetuities (g=0)	$Ku_i - \frac{TS_i}{V^L_{i-1}}$	$Ku_i - \frac{Ku_i TD_{i-1}}{V^L_{i-1}}$
Growing perpetuities (g different from 0)	$Ku - \frac{TS}{V^L}$	$Ku_i - \frac{(Ku_i - g)TD\%Kd}{(Kd - g)}$
Finite	$Ku_i - \frac{TS_i}{V^L_{i-1}}$	$Ku_i - \frac{TS_i}{V^L_{i-1}} - (Ku_i - Kd_i)\frac{V^{TS}_{i-1}}{V^L_{i-1}}$

Applied to the CCF

Table 6. Traditional WACC formula for the CCF, $WACC^{CCF}$, according to the Discount rate for TS

	$\psi_i = Ku_i$	$\psi_i = Kd_i$
Perpetuity and Finite	$\frac{Kd_i D_{i-1}}{V^L_{i-1}} + \frac{Ke_i E_{i-1}}{V^L_{i-1}}$	$\frac{Kd_i D_{i-1}}{V^L_{i-1}} + \frac{Ke_i E_{i-1}}{V^L_{i-1}}$

The general formula for the $WACC^{CCF}$ is as follows.

$$WACC_{adjusted} = Ku_i - \left(Ku_i - \psi_i\right)\frac{V^{TS}_{i-1}}{V^L_{i-1}} \qquad (10)[4]$$

Table 7. Adjusted $WACC^{CCF}$ formula according to the discount rate for TS

	$\psi_i = Ku_i$	$\psi_i = Kd_i$
Perpetuities	Ku_i	$Ku_i - \left(Ku_i - Kd_i\right)\frac{TD}{V^L_{i-1}}$
Finite	Ku_i	$Ku_i - \left(Ku_i - Kd_i\right)\frac{V^{TS}_{i-1}}{V^L_{i-1}}$

For a detailed derivation of these formulations see Appendix A, Vélez–Pareja and Tham, 2001 and Tham and Vélez–Pareja 2003.

[4] See appendix A for derivation.

When the traditional WACC and the adjusted WACC can be used? It depends on what happens to the tax savings. There are situations when the tax savings cannot be earned in full a given year due to a very low Earnings Before Interest and Taxes, EBIT, (and there exist legal provision for losses carried forward, LCF) or the tax savings are not earned in the current year because taxes are not paid the same year as accrued or there are other sources different from interest charges that generate tax savings, such as adjustments for and inflation to the financial statements. (See Vélez–Pareja and Tham, 2003 and Tham and Vélez–Pareja 2003). When these anomalies occur the traditional formulation for the traditional WACC cannot be used. This is shown in a simple table below:

Table 8. Conditions for the use of the two versions of WACC

WACC	Conditions
Traditional WACC $Kd_i(1-T)\frac{D_{i-1}}{V^L_{i-1}}+\frac{Ke_iE_{i-1}}{V^L_{i-1}}$	Taxes paid in the same period as accrued. Enough EBIT to earn the TS. When the only source of TS is the interest paid.
Adjusted WACC $WACC_{adjusted}=Ku_i-\frac{TS_i}{V^L_{i-1}}-[(Ku_i-\psi_i)\frac{V^{TS}_{i-1}}{V^L_{i-1}}$	For any situation. However, it is mandatory when taxes are not paid the same year as accrued and there is not enough EBIT to earn the TS. When there are other sources of TS.

Section Four

Relationship between Terminal Value for FCF and Terminal Value for CFE

Case a) Considering only the outstanding debt at year N.

As we mention in Section One the market value for the levered equity is a residual value. This applies to any point in time. This is, as in stated in equation (8)

Market equity value = Total Levered value – Debt (8)

When this residual relationship is applied to the terminal value, we have

TV for equity = TV for the FCF (for the firm) − Debt (11)

This means that we do not need to calculate a growth rate (G) for the CFE. However, we can derive it departing from the basic residual relationship[5]:

$$TV_{equity} = \frac{CFE(1+G)}{K_e - G} = \frac{FCF(1+g)}{WACC_{perp} - g} - D \tag{12}$$

We can solve this expression for G and that would be the consistent growth rate for the CFE (consistency defined as above).

Multiplying by Ke − G we have

$$CFE(1+G) = \left(\frac{FCF(1+g)}{WACC_{perp} - g} - D\right)(K_e - G) \tag{13}$$

Reorganizing terms, we have

$$CFE = \left(\frac{FCF(1+g)}{WACC_{perp} - g} - D\right)(K_e - G) - CFE \times G \tag{14}$$

Grouping terms with G

$$CFE - \left(\frac{FCF(1+g)}{WACC_{perp} - g} - D\right)K_e = -G\left(\frac{FCF(1+g)}{WACC_{perp} - g} - D\right) - CFE \times G \tag{15}$$

Reorganizing the right hand side and isolating G

$$CFE - \left(\frac{FCF(1+g)}{WACC_{perp} - g} - D\right)K_e = G\left(-\left(\frac{FCF(1+g)}{WACC_{perp} - g} - D\right) - CFE\right) \tag{16}$$

Solving for G

[5] In this text we are assuming that the TV is calculated from the FCF just for keeping simple the formulation. If we wish to keep FCF increasing we have to include some additional investment that takes into account the depreciation that maintains the level of assets and just keeps the FCF constant and an additional fraction of the FCF to grant the growth for perpetuity. This is done using the Net operating profit less adjusted taxes. NOPLAT as a measure of the FCF (this keeps the NOPLAT constant because we are investing the depreciation) and subtracting a fraction of it related to the return on invested capital, ROIC, (fraction = g/ROIC) to grant growth.

$$G=\frac{CFE-\left(\frac{FCF(1+g)}{WACC_{perp}-g}-D\right)K_e}{\left(-\left(\frac{FCF(1+g)}{WACC_{perp}-g}-D\right)-CFE\right)} \tag{17}$$

Simplifying

$$G=\frac{CFE\left(WACC_{perp}-g\right)-\left[FCF(1+g)-D\left(WACC_{perp}-g\right)\right]K_e}{-FCF(1+g)+D\left(WACC_{perp}-g\right)-CFE\left(WACC_{perp}-g\right)} \tag{18}$$

Multiplying by −1 the denominator and the numerator and ordering terms we have

$$G=\frac{FCF(1+g)K_e-DK_e\left(WACC_{perp}-g\right)-CFE\left(WACC_{perp}-g\right)}{FCF(1+g)-D\left(WACC_{perpetuity}-g\right)+CFE(WACC_{perp}-g)} \tag{19}$$

This is an awful formula, but consistent. If we set D equal to zero, then CFE is identical to the FCF and Ke is identical to the WACC and our equation (19) results in

$$\begin{aligned}
G&=\frac{FCF(1+g)WACC_{perp}-FCF\left(WACC_{perp}-g\right)}{FCF(1+g)+FCF(WACC_{perpetuity}-g)}\\
&=\frac{FCF\times WACC_{perp}+FCF\times g\times WACC_{perp}-FCF\times WACC_{perp}+FCF\times g}{FCF+FCF\times g+FCF\times WACC_{perp}-FCF\times g}\\
&=\frac{+FCF\times g\times WACC_{perp}+FCF\times g}{FCF+FCF\times WACC_{perp}}\\
&=\frac{+FCF\times g(1+WACC_{perp})}{FCF(1+WACC_{perp})}\\
&=g
\end{aligned} \tag{20}$$

As expected when D = 0. This is the growth rate for the FCF is the same as the growth rate for the CFE.

Case b) Considering the level of debt to obtain the perpetual leverage $D\%_{perp}$ and using the CFE for N.

We can consider the total debt at year N instead of the debt outstanding from the forecasting period. This is we can consider the constant leverage for perpetuity, $D\%_{perp}$ as the basis of our analysis. In this case we have to separate the regular projected CFE and the extra CFE generated by the adjustment of the current debt to reach the desired $D\%_{perp}$. As we mention in Section One the market value for the levered equity is a residual value. This applies to any point in time. This is, as in stated in equation (8) but we write that equation in terms of $D\%_{perp}$ for year N as

Market equity value = Total Levered value – $D\%_{perp} \times TV$ (8)

When this residual relationship is applied to the terminal value (TV), we have

TV for equity = TV with the NOPLAT (for the firm) × $(1 - D\%_{perp})$ (21)[6]

This means that we do not need to calculate a growth rate (G) for the CFE. However, we can derive it departing from the basic residual relationship between values in equation (21):

$$TV_{equity} = \frac{CFE(1+G)}{K_e - G} = \frac{NOPLAT(1+g)}{WACC_{perp} - g}\left(1 - D\%_{perp}\right) \qquad (22)^{7}$$

Where $WACC_{perp}$ and $D\%_{perp}$ are the constant WACC and the leverage we define for perpetuity.

We can solve this expression for G and that would be the consistent growth rate for the CFE (consistency defined as above).

Multiplying by Ke − G we have

$$CFE(1+G) = \left(\frac{FCF(1+g)}{WACC_{perp} - g}\left(1 - D\%_{perp}\right)\right)\left(K_e - G\right) \qquad (23)$$

Reorganizing terms, we have

$$CFE = \left(\frac{FCF(1+g)}{WACC_{perp} - g}\left(1 - D\%_{perp}\right)\right)\left(K_e - G\right) - CFE \times G \qquad (24)$$

Grouping terms with G

$$CFE-\left(\frac{FCF(1+g)}{WACC_{perp}-g}\left(1-D\%_{perp}\right)\right)K_e=-G\left(\frac{FCF(1+g)}{WACC_{perp}-g}\left(1-D\%_{perp}\right)\right)-CFE\times G \quad (25)$$

Reorganizing the right hand side and isolating G

$$CFE-\left(\frac{FCF(1+g)}{WACC_{perp}-g}\left(1-D\%_{perp}\right)\right)K_e= \\ G\left(-\left(\frac{FCF(1+g)}{WACC_{perp}-g}\left(1-D\%_{perp}\right)\right)-CFE\right) \quad (26)$$

Solving for G

$$G=\frac{CFE-\left(\frac{FCF(1+g)}{WACC_{perp}-g}\left(1-D\%_{perp}\right)\right)K_e}{\left(-\left(\frac{FCF(1+g)}{WACC_{perp}-g}\left(1-D\%_{perp}\right)\right)-CFE\right)} \quad (27)$$

Simplifying

$$G=\frac{CFE\left(WACC_{perp}-g\right)-\left[FCF(1+g)\left(1-D\%_{perp}\right)\right]K_e}{-FCF(1+g)\left(1-D\%_{perp}\right)-CFE\left(WACC_{perp}-g\right)} \quad (28)$$

Multiplying by −1 the denominator and the numerator and ordering terms we have

$$G=\frac{FCF(1+g)\left(1-D\%_{perp}\right)K_e-CFE\left(WACC_{perp}-g\right)}{FCF(1+g)\left(1-D\%_{perp}\right)+CFE(WACC_{perp}-g)} \quad (29)$$

This is also an awful formula, but consistent. If we set $D\%_{perp}$ equal to zero, then CFE is identical to the FCF and Ke is identical to the WACC and our equation (29) results in

$$G = \frac{FCF(1+g)WACC_{perp} - FCF(WACC_{perp} - g)}{FCF(1+g) + FCF(WACC_{perp} - g)}$$

$$= \frac{FCF \times WACC_{perp} + FCF \times g \times WACC_{perp} - FCF \times WACC_{perp} + FCF \times g}{FCF + FCF \times g + FCF \times WACC_{perp} - FCF \times g}$$

$$= \frac{+ FCF \times g \times WACC_{perp} + FCF \times g}{FCF + FCF \times WACC_{perp}} \tag{30}$$

$$= \frac{+ FCF \times g(1 + WACC_{perp})}{FCF(1 + WACC_{perp})}$$

$$= g$$

As expected when $D\%_{perp} = 0$. This is the growth rate for the FCF is the same as the growth rate for the CFE.

We could proceed the other way around and calculate the growth for the CFE and derive in a similar fashion g, the growth rate for the FCF. In any case, equation (11) has to be validated.

As a bottom line, the growth rate for the FCF, g is different than the growth rate for the CFE, G when we have a change in the level of debt to obtain the desired perpetual leverage, $D\%_{perp}$.

When using this approach we have to take into account that the extra debt (or the repayment of debt to reach the desired $D\%_{perp}$) is a negative (positive) flow in the CFD and a corresponding positive (negative) flow in the CFE. This amount has to be added (subtracted) to the TV for the CFE.

Debt value at N = DO at N + New debt (31a)

And new debt ND is

ND = Repurchase of equity = $D\%_{perp}$ × (TV for CFE + Repurchase of equity + DO at N) − DO at N (31b)

Solving for Repurchase of equity, RE, we have

$RE - D\%_{perp}\, RE = D\%_{perp} \times (\text{TV for CFE} + \text{DO at N}) - \text{DO at N}$ (31c)

$RE\,(1 - D\%_{perp}) = D\%_{perp} \times (\text{TV for CFE} + \text{DO at N}) - \text{DO at N}$ (31d)

$RE = (D\%_{perp} \times (\text{TV for CFE} + \text{DO at N}) - \text{DO at N})/(1 - D\%_{perp})$ (31e)

But

New Debt = Repurchase of equity (31f)

The new equity at year N will be

Equity at the end of year N = RE + DO (31g)

We have to mention that both approaches, using the growth for CFE, G, with the debt outstanding or with the perpetual leverage, $D\%_{perp}$ the results are exactly the same. This can be seen in Appendix B where we present a numerical example.

In the same line of thought we wish to show that we can use g as the growth rate for other cash flows, such as the CFD and the TS.

If we set a perpetual level of leverage it means that the relationship between the value of debt and total levered value using a growing perpetuity is constant and equal to $D\%_{perp}$, the leverage in perpetuity. This is,

$$D\%_{perp} = \frac{\dfrac{CFD(1+G_D)}{Kd - G_D}}{\dfrac{FCF(1+g)}{WACC_{perp} - g}} \tag{32}$$

Organizing the fractions,

$$D\%_{perp} = \frac{(WACC_{perp} - g)CFD(1+G_D)}{FCF(1+g)(Kd - G_D)} \tag{33}$$

$$D\%_{perp}\left(FCF(1+g)(Kd - G_D)\right) = (WACC_{perp} - g)CFD(1+G_D) \tag{34}$$

Developing the operations in (34)

$$D\%_{perp}\left(FCF(1+g)Kd - G_D FCF(1+g)\right) \\ = (WACC_{perp} - g)CFD + G_D(WACC_{perp} - g)CFD \tag{35}$$

$$D\%_{perp} FCF(1+g)Kd - D\%_{perp} G_D FCF(1+g) \\ = (WACC_{perp} - g)CFD + G_D(WACC_{perp} - g)CFD \tag{36}$$

Collecting terms with D% to the right hand side of the equation

$$D\%_{perp} FCF(1+g)Kd - (WACC_{perp} - g)CFD \\ = D\%_{perp} G_D FCF(1+g) + G_D(WACC_{perp} - g)CFD \tag{37}$$

Factorizing G_D

$$D\%_{perp} FCF(1+g)Kd - (WACC_{perp} - g)CFD \\ = G_D\left(D\%_{perp} FCF(1+g) + (WACC_{perp} - g)CFD\right) \tag{38}$$

Solving for G_D

$$G_D = \frac{D\%_{perp} FCF(1+g)Kd - (WACC_{perp} - g)CFD}{D\%_{perp} FCF(1+g) + (WACC_{perp} - g)CFD} \tag{39}$$

But we have to remember that CFD is the CFD at period N. Debt at period N is

$$D = D\% \frac{FCF(1+g)}{WACC_{perp} - g} \tag{40}$$

and CFD at period N+1 is

$$(Kd - g)D\% \frac{FCF(1+g)}{WACC_{perp} - g} \tag{41}$$

Hence, CFD at period N is

$$\frac{(Kd - g)D\% \dfrac{FCF(1+g)}{WACC_{perp} - g}}{1+g} \tag{42}$$

and the G_D is then

$$G_D = \frac{D\%_{perp}FCF(1+g)Kd - (WACC_{perp} - g)\dfrac{\left[(Kd - g)D\%\dfrac{FCF(1+g)}{WACC_{perp} - g}\right]}{1+g}}{D\%_{perp}FCF(1+g) + (WACC_{perp} - g)\dfrac{\left[(Kd - g)D\%\dfrac{FCF(1+g)}{WACC_{perp} - g}\right]}{1+g}} \tag{43}$$

Developing the second term in numerator and denominator we have

$$G_D = \frac{D\%_{perp}FCF(1+g)Kd - \dfrac{(Kd - g)D\%FCF(1+g)}{1+g}}{D\%_{perp}FCF(1+g) + \dfrac{(Kd - g)D\%FCF(1+g)}{1+g}} \tag{44}$$

Dividing by $D\%_{perp}FCF(1+g)$ and simplifying we have

$$G_D = \frac{Kd - \dfrac{(Kd - g)}{1+g}}{1 + \dfrac{(Kd - g)}{1+g}} \tag{45}$$

Simplifying,

$$G_D = \frac{Kd(1+g) - (Kd - g)}{1 + g + (Kd - g)} \tag{46}$$

and

$$G_D = \frac{Kdg + g}{1 + Kd} = g \tag{47}$$

Section Five

Starting the Valuation Process and Calculating Ke or Ku as Departing Point

When we value a firm we have to estimate the value of Ke or the value of Ku. See Vélez–Pareja (2003).

Assume we start with Ku and that the proper discount rate for TS is Ku. Then we start calculating Ku using some of the methods presented in Vélez–Pareja (2003) or Tham and Vélez–Pareja (2003): subjectively, unlevering some betas from the market or unlevering the accounting beta for the firm. For instance, if we unlever the beta for some firms, we can use CAPM and we calculate Ku.

$$\beta_u = \frac{\beta_{lev}}{1 + \frac{D_t}{E_t}} \tag{48}$$

where t stands for traded.

Then we calculate Ku as

$$Ku = R_f + \beta u(Rm - Rf) \tag{49}$$

With this Ku, we calculate Ke as

$$Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} \tag{50}$$

And we can calculate the adjusted WACC as

$$WACC_{adjusted} = Ku_i - \frac{TS_i}{V^L_{i-1}} \tag{51}$$

Or the traditional WACC as

$$WACC = Kd_i(1-T)\frac{D_{i-1}}{V^L_{i-1}} + \frac{Ke_i E_{i-1}}{V^L_{i-1}} \tag{52}$$

Now assume we start with Ke and that the proper discount rate for TS is Ku.

We calculate Ke using some of the methods presented in Vélez–Pareja (2003) or Tham and Vélez–Pareja (2003): subjectively, unlevering and levering some betas from the market or using the accounting beta for the firm. For instance, if we calculate the accounting beta for the firm, we can use CAPM and we calculate Ke.

$$Ke = Rf + \beta_{acc}(Rm - Rf) \tag{53}$$

With this Ke we calculate Ku unlevering the β_{acc} as

$$\beta_u = \frac{\beta_{acc}}{1 + \frac{D}{E}} \tag{54}$$

With this βu we calculate Ku as Ku = Rf + βu(Rm − Rf) and we can calculate the adjusted WACC as

$$\text{WACC}_{\text{adjusted}} = \text{Ku}_i - \frac{\text{TS}_i}{\text{V}_{i-1}^{L}} \tag{55}$$

Or with the Ke we calculate the traditional WACC as

$$\text{WACC} = \text{Kd}_i(1-\text{T})\frac{\text{D}_{i\text{-}1}}{\text{V}_{i\text{-}1}^{L}} + \frac{\text{Ke}_i \text{E}_{i\text{-}1}}{\text{V}_{i\text{-}1}^{L}} \tag{56}$$

If we start we Ke directly, say, subjectively, we calculate Ku unlevering the beta for Ke as

$$\beta_{\text{sub}} = \frac{\text{Ke}_{\text{sub}} - \text{Rf}}{\text{Rm} - \text{Rf}} \tag{57}$$

and unlever β_{sub} as

$$\beta_u = \frac{\beta_{\text{sub}}}{1 + \dfrac{\text{D}}{\text{E}}} \tag{58}$$

and calculate Ku as

$$\text{Ku} = \text{Rf} + \beta\text{u}(\text{Rm} - \text{Rf}) \tag{59}$$

With Ku we calculate the adjusted WACC as described above.

When we use Ke to derive from it the Ku, we find circularity and that is a problem. The circularity appears because we have to unlever the Ke with the market value of the firm and that is what we need to calculate. When we use Ku, it is straightforward.

We cannot have BOTH, Ku and Ke as independent variables. This is, either we define Ku and we derive Ke using that Ku or we define Ke and from there we derive Ku. In the valuation of a firm (or a project) we cannot say that we have as inputs Ku AND Ke at the same time. Given one of them, the other is defined as mentioned above.

Section Six

Concluding Remarks

We have presented a summary of the proper relationships for cash flows, terminal values, cost of capital and a procedure to start the valuation process using Ke or Ku, the cost of levered equity or the cost of unlevered equity, respectively.

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Appendix A

This appendix is based on Tham, Joseph and Ignacio Velez–Pareja, 2002, An Embarrassment of Riches: Winning Ways to Value with the WACC, Working Paper in SSRN, *Social Science Research Network*.

General expression for the return to levered equity Ke_i

We briefly derive the general algebraic expressions for the cost of capital that is applied to finite cash flows. First, we show that in general, the return to levered equity Ke_i is a function of ψ_i, and this is a most important point. Second, we derive the general expressions for the WACCs.

First, we write the main equations as follows.

$$V^{Un}_{i-1} \times (1 + Ku_i) - V^{Un}_i = FCF_i \quad (A1)$$

$$E^L_{i-1} \times (1 + Ke_i) - E^L_i = CFE_i \quad (A2)$$

$$D_{i-1} \times (1 + Kd_i) - D_i = CFD_i \quad (A3)$$

$$V^{TS}_{i-1} \times (1 + \psi_i) - V^{TS}_i = TS_i \quad (A4)$$

We know that,

$$FCF_i + TS_i = CFE_i + CFD_i \quad (A5a)$$

and

$$V^{Un}_i + V^{TS}_i = E^L_i + D_i \quad (A5b)$$

and

$$V^{Un}_i = E^L_i + D_i - V^{TS}_i \quad (A5c)$$

To obtain the general expression for the Ke, substitute equations A1 to A4 into equation A5a.

$$V^{Un}_{i-1} \times (1 + Ku_i) - V^{Un}_i + V^{TS}_{i-1} \times (1 + \psi_i) - V^{TS}_i$$

$$= E^L_{i-1} \times (1 + Ke_i) - E^L_i + D_{i-1} \times (1 + Kd_i) - D_i \quad (A6)$$

We simplify applying (5b) and obtain,

$$V^{Un}_{i-1} \times Ku_i + V^{TS}_{i-1} \times \psi_i = E^{L}_{i-1} \times Ke_i + D_{i-1} \times Kd_i \qquad \text{(A7)}$$

Solve for the return to levered equity and using A5c.

$$E^{L}_{i-1} \times Ke_i = (E^{L}_{i-1} + D_{i-1} - V^{TS}_{i-1}) \times Ku_i + V^{TS}_{i-1} \times \psi_i - D_{i-1} \times Kd_i \qquad \text{(A8)}$$

Collecting terms and rearranging, we obtain,

$$E^{L}_{i-1} \times Ke_i = E^{L}_{i-1} \times Ku_i + (Ku_i - Kd_i) \times D_{i-1} - (Ku_i - \psi_i) \times V^{TS}_{i-1} \qquad \text{(A9)}$$

Solving for the return to levered equity, we obtain,

$$Ke_i = Ku_i + (Ku_i - Kdi)\frac{D_{i-1}}{E^{L}_{i-1}} - (Ku_i - \psi i)\frac{V^{TS}_{i-1}}{E^{L}_{i-1}} \qquad \text{(A10)}$$

Adjusted WACC applied to the FCF

We can express the FCF as follows

$$FCFi = Ku \times V^{Un}_{i-1} = WACC_i \times V^{L}_{i-1} \qquad \text{(A11)}$$

Let $WACC^{Adj}_i$ be the adjusted WACC that is applied to the FCF in year i.

$$V^{L}_{i-1} \times WACC^{Adj}_i = D_{i-1} \times Kd_i - TS_i + E^{L}_{i-1} \times Ke_i \qquad \text{(A12)}$$

$$V^{L}_{i-1} \times WACC^{Adj}_i = V^{Un}_{i-1} \times Ku_i + V^{TS}_{i-1} \times \psi_i - TS_i \qquad \text{(A13)}$$

$$V^{L}_{i-1} \times WACC^{Adj}_i = (V^{L}_{i-1} - V^{TS}_{i-1}) \times Ku_i + V^{TS}_{i-1} \times \psi_i - TS_i \qquad \text{(A14)}$$

$$V^{L}_{i}\text{-}1 \times WACC^{Adj}_i = V^{L}_{i-1} \times Ku_i - (Ku_i - \psi_i) \times V^{TS}_{i-1} - TS_i \qquad \text{(A15)}$$

Solving for the WACC in equation A15, we obtain,

$$WACC^{Adj}_i = Ku_i - (Ku_i - \psi)\frac{V^{TS}_{i-1}}{V^{L}_{i-1}} - \frac{TS_i}{V^{L}_{i-1}} \qquad \text{(A16)}$$

Adjusted WACC applied to the CCF

We know that the CCF is equal to the sum of the FCF and the TS.

$$CCF_i = CFD_i + CFE_i = FCF_i + TS_i \qquad \text{(A17)}$$

Let $WACC^{Adj}_i$ be the adjusted WACC applied to the CCF.

$$V^{L}_{i-1} \times WACC^{Adj}_i = V^{Un}_{i-1} \times Ku_i + V^{TS}_{i-1} \times \psi_i \qquad \text{(A18)}$$

$$V^{L}_{i-1} \times WACC^{Adj}_{i} = V^{L}_{i-1} \times Ku_i - (Ku_i - \psi_i) \times V^{TS}_{i-1} \quad \text{(A19a)}$$

Solving for the WACC, we obtain,

$$WACC^{Adj}_{i} = Ku_i - (Ku_i - \psi_i)\frac{V^{TS}_{i-1}}{V^{L}_{i-1}} \quad \text{(A19b)}$$

Cash flows in perpetuity

Derivation of the adjusted WACC

Assume that the growth rate g > 0 and the CF are in perpetuity. The debt is risk-free, that is Kd = Rf and the leverage θ is constant.

The adjusted WACC applied to the FCF is $WACC_{perp}$.

$$V^{L} = \frac{FCF}{WACC_{perp} - g} \quad \text{(A20a)}$$

$$FCF = V^{L}(WACC_{perp} - g) \quad \text{(A20b)}$$

$$V^{Un} = \frac{FCF}{Ku - g} \quad \text{(A21a)}$$

$$FCF = V^{Un}(Ku - g) \quad \text{(A21b)}$$

$$VTS = \frac{TS}{\psi - g} \quad \text{(A22a)}$$

$$TS = V^{TS}(\psi - g) \quad \text{(A22b)}$$

Where g is the growth rate for TS (and CFD).

Equating equations A20b and A21b, we obtain,

$$V^{L}(WACC_{perp} - g) = V^{Un}(Ku - g) \quad \text{(A23a)}$$

$$WACC_{perp}V^{L} = KuV^{Un} - g(V^{L} - V^{Un}) \quad \text{(A23b)}$$

$$WACC_{perp}V^{L} = Ku(V^{L} - V^{TS}) - g(V^{L} - V^{Un}) \quad \text{(A23c)}$$

$$WACC_{perp}\ V^{L} = KuV^{L} - KuV^{TS} - gV^{TS} \quad \text{(A23d)}$$

$$WACC_{perp}\, V^L = KuV^L - KuV^{TS} - gV^{TS} \tag{A23e}$$

$$WACC_{perp} = Ku - \frac{KuV^{TS}}{V^L} - \frac{gV^{TS}}{V^L} \tag{A23f}$$

$$WACC_{perp} = Ku - \frac{(Ku - g)V^{TS}}{V^L} \tag{A23g}$$

There are four cases.

Case 1a: $g = 0$ and $\psi = Ku$

$$WACC_{perp} = Ku - \frac{KuV^{TS}}{V^L} = Ku - \frac{TS}{V^L} = Ku - \frac{TDKd}{V^L} = Ku - TKdD\%_{perp} \tag{A24a}$$

Case 1b: $g = 0$ and $\psi = Kd$

$$WACC_{perp} = Ku - \frac{KuV^{TS}}{V^L} = Ku - \frac{KuTD}{V^L} = Ku\left(1 - \frac{TD}{V^L}\right) \tag{A24b}$$

Case 2a: $g > 0$ and $\psi = Ku$

$$WACC_{perp} = Ku - \frac{(Ku - g)V^{TS}}{V^L} = Ku - \frac{TS}{V^L} = Ku - \frac{TDKd}{V^L} \tag{A25a}$$

Case 2b: $g > 0$ and $\psi = Kd$

$$WACC_{perp} = Ku - \frac{(Ku - g)V^{TS}}{V^L} = Ku - \frac{\frac{(Ku - g)TS}{Kd - g}}{V^L} \tag{A25b}$$

$WACC_{perp} = Ku - [(Ku - g)/(Kd - g)]TDKd/V^L$

$$WACC_{perp} = Ku - \frac{(Ku - g)TDKd}{(Kd - g)V^L} \tag{A25c}$$

$$WACC_{perp} = Ku_i - \frac{(Ku_i - g)TD\%Kd}{(Kd - g)} \quad \text{(A25d)}$$

Appendix B

In this appendix we show an example to illustrate the ideas presented in the paper.

Case a) Considering only the outstanding debt at year N.

Assume we have the following information:

1. The cost of debt, Kd is constant and equal to 13%. Constant rates for cost of debt are a simplification; in reality that cost is not constant for several reasons. One of them is that the firm might have several sources of financial debt with different rates. The correct way to consider the cost of debt is to divide the interest charges at the end of the period by the debt balance at the beginning of the period.
2. The risk free rate, Rf, is 10%.
3. The market risk premium MRP is 5%.
4. The unlevered beta βu is 1.01875.
5. The tax rate is 40% and taxes are paid the same year as accrued. There is enough EBIT to earn the TS.
6. The growth rate for the FCF is 7%.
7. The desired or expected leverage in perpetuity, constant, is 50%
8. In addition we know that the debt balance and the CFE is as follows

Table A1. Debt balance and CFE

Year	2003	2004	2005	2006	2007	2008
Debt balance	23.08	30.77	38.46	46.15	46.15	46.15
CFE		14.09	16.49	17.49	10.20	11.20

The CFE can be derived from the cash budget looking at the dividends and/or repurchase of equity and/or new equity investment.

From this information we can make some estimates, as follows:

1. We can estimate the unlevered cost of equity, Ku, as follows, using the CAPM

$Ku = Rf + \beta uMRP = 10\% + 1.01875 \times 5\% = 15.094\%$

2. The interest payments and the TS can be calculated. The interest charge is $I = Kd \times D_{t-1}$ and the TS is $T \times I$. In fact the interest charges can de read directly from the debt schedule or the cash budget.

Table A2. Interest charges and tax savings

Year	2003	2004	2005	2006	2007	2008
Interest charges		3.00	4.00	5.00	6.00	6.00
TS		1.20	1.60	2.00	2.40	2.40

3. The CFD can be calculated from the debt balance and the interest charges. The principal payment is the difference between two successive debt balances, $PPMT_t = D_{t-1} - D_t$. In fact, the PPMT can be read directly from the debt schedule or the cash budget.

Table A3. Debt balance, principal payment, interest and CFD

Year	2003	2004	2005	2006	2007	2008
Debt balance	23.08	30.77	38.46	46.15	46.15	46.15
PPMT		–7.69	–7.69	–7.69	0.00	0.00
Interest charges I		3.00	4.00	5.00	6.00	6.00
CFD = PPMT + I		−4.69	−3.69	−2.69	6.00	6.00

4. For year N we have two cases: with new debt to reach the D% at perpetuity in the CFD and without that new debt. With this we have the CFE, the CFD and the TS, hence we can estimate the FCF and the CCF as follows:

In the first case we have

Table A4a. FCF and CCF derived from CFD, CFE and TS for year N

Year	2003	2004	2005	2006	2007	2008
Debt	23.08	30.77	38.46	46.15	46.15	46.15
New debt						D% × TV − 46.15
Total debt	23.08	30.77	38.46	46.15	46.15	D% × TV
Interest charges		3.00	4.00	5.00	6.00	6.00
Principal payment		–7.69	–7.69	–7.69	0.00	46.15 − D% × TV
Total CFD		−4.69	−3.69	−2.69	6.00	6.00 + 46.15 − D% × TV
CFE		14.09	16.49	17.49	10.20	11.20
Repurchase of equity						D% × TV − 46.15
Total CFE		14.09	16.49	17.49	10.20	11.20 + D% × TV − 46.15
Sum = CFD + CFE		9.40	12.80	14.80	16.20	17.20
TS		1.20	1.60	2.00	2.40	2.40
FCF = CFD + CFE – TS		8.20	11.20	12.80	13.80	14.80
CCF = CFD + CFE		9.40	12.80	14.80	16.20	17.20

For the second case we have

Table A4b Cash flows not taking into account the new debt and equity repurchase for year N

	2003	2004	2005	2006	2007	2008
Debt at beginning	23.08	30.77	38.46	46.15	46.15	46.15
Interest charges		3.00	4.00	5.00	6.00	6.00
TS		1.20	1.60	2.00	2.40	2.40
Principal payment		–7.69	–7.69	–7.69	0.00	0.00
CFD		(4.69)	(3.69)	(2.69)	6.00	6.00
CFE		14.09	16.49	17.49	10.20	11.20
Sum		9.40	12.80	14.80	16.20	17.20
FCF = CFD + CFE – TS		8.20	11.20	12.80	13.80	14.80
CCF = CFD + CFE		9.40	12.80	14.80	16.20	17.20

Observe that the FCF and the CCF are the same no matter which the new debt is and that the values are independent from including or not the new debt in the analysis.

Finite cash flows and Kd as the discount rate for the TS

Now we will derive the firm and equity values using several methods and assuming finite cash flows and Kd as the discount rate for the TS.

In order to calculate the terminal value TV, for the FCF we have to estimate the WACC at perpetuity $WACC_{perp}$, given our assumptions. From equation A26d we have

$$WACC_{perp} = Ku_i - \frac{(Ku_i - g)TD\%Kd}{(Kd - g)}$$

And then $WACC_{perp}$ is

$$WACC_{perp} = 15.09\% - \frac{(15.09\% - 7\%) \times 40\% \times 50\% \times 13\%}{(13\% - 7\%)} = 11.59\%$$

On the other hand, the Ke_{perp} is

$$Ke_{perp} = Ku_i + (Ku_i - Kd_i)(1 - T)\frac{D_{i-1}}{E^L_{i-1}}$$

In our example we have

$$Ke_{perp} = 15.09\% + (15.09\% - 13\%.00)(1 - 40.0\%)\frac{50\%}{1 - 50\%} = 16.35\%$$

On the other hand, the terminal value for the FCF is

$$TV_{FCF} = \frac{FCF(1+g)}{WACC_{perp} - g}$$

$$TV_{FCF} = \frac{FCF(1+g)}{WACC_{perp} - g} = \frac{14.80(1+7\%)}{11.59\% - 7\%} = 345{,}28$$

Using the $WACC_{adj}$, $Ku_i - \frac{TS_i}{V^L_{i-1}} - (Ku_i - Kd_i)\frac{V^{TS}_{i-1}}{V^L_{i-1}}$ we calculate solving the circularity, the levered value of the firm and the equity. The present value of the TS is calculated using Kd as the discount rate. Using (38) to calculate WACC with circularity for the finite cash flows, we find

Table A5. ψ=Kd: Levered values calculated with FCF and Adjusted WACC

Year		2004	2005	2006	2007	2008
FCF		8.20	11.20	12.80	13.80	14.80
TV						345.28
FCF with TV		8.20	11.20	12.80	13.80	360.08
TS		1.20	1.60	2.00	2.40	2.40
Value of TS	6.48	6.12	5.31	4.00	2.12	
$WACC_{adj}$		14.48%	14.37%	14.29%	14.23%	14.32%
Total levered value	216.6096	239.7686	263.0305	287.8205	314.9796	
Levered equity	193.5327	208.9993	224.5690	241.6666	268.8257	

Now using the CCF and the $WACC^{CCF}$ we calculate the same values. From table 7 we know that the adjusted $WACC^{CCF}$ is $Ku_i - (Ku_i - Kd_i)\frac{V_{i-1}^{TS}}{V_{i-1}^{L}}$

Table A6. ψ=Kd: Levered values calculated with CCF and WACC for CCF

Year	2003	2004	2005	2006	2007	2008
CCF = FCF + TS = CFD + CFE		9.40	12.80	14.80	16.20	362.48
TS		1.20	1.60	2.00	2.40	2.40
Value of TS	6.48	6.12	5.31	4.00	2.12	
$WACC^{CCF} = Ku_i - (Ku_i - Kd_i)\frac{V_{i-1}^{TS}}{V_{i-1}^{L}}$		15.03%	15.04%	15.05%	15.06%	15.08%
Total levered value	216.6096	239.7686	263.0305	287.8205	314.9796	
Levered equity	193.5327	208.9993	224.5690	241.6666	268.8257	

As it should be, the values for the firm and equity match.

As the conditions required for using the traditional WACC formulation and the FCF are fulfilled, we present the values calculated using it. From Table 3 we know that the correct formulation for Ke with finite cash flows and Kd as the discount rate for TS is $Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E_{i-1}^{L}} - (Ku_i - Kd_i)\frac{V_{i-1}^{TS}}{E_{i-1}^{L}}$ and the traditional WACC is

$$Kd_i(1-T)\frac{D_{i-1}}{V_{i-1}^{L}} + \frac{Ke_i E_{i-1}}{V_{i-1}^{L}}$$

Table A7. ψ=Kd: Levered values calculated with FCF and traditional WACC

Year	2003	2004	2005	2006	2007	2008
FCL		8,20	11,20	12,80	13,80	14,80
TV						345,28
FCF + TV_{FCF}		8.2	11.2	12.8	13.8	360.1
Kd(1−T)		7.80%	7.80%	7.80%	7.80%	7.80%
Kd(1−T)D%		0.83%	1.00%	1.14%	1.25%	1.14%
TS		1.20	1.60	2.00	2.40	2.40
Value of TS	6.48	6.12	5.31	4.00	2.12	
Ke = $Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} - (Ku_i - Kd_i)\frac{V^{TS}_{i-1}}{E^L_{i-1}}$		15.27%	15.34%	15.40%	15.46%	15.44%
KeE%		13.65%	13.37%	13.15%	12.98%	13.17%
Traditional WACC		14.48%	14.37%	14.29%	14.23%	14.32%
Total levered value	216.6096	239.7686	263.0305	287.8205	314.9796	
Levered equity	193.5327	208.9993	224.5690	241.6666	268.8257	

Again, as expected, the calculated values match.

Now we calculate the TV for the CFE as the TV for the FCF minus the outstanding debt and the equity value (and total levered value) using the CFE.

Table A8 ψ=Kd: Calculation of levered values using TV_{CFE} as TV_{FCF} minus debt

	2003	2004	2005	2006	2007	2008
CFE		14.09	16.49	17.49	10.20	11.20
$TV_{CFE} = VT_{FCF} - \text{debt}$						299.12
CFE + VT_{CFE}		14.09	16.49	17.49	10.20	310.32
TS		1.20	1.60	2.00	2.40	2.40
Value of TS	6.48	6.12	5.31	4.00	2.12	
Ke= $Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} - (Ku_i - Kd_i)\frac{V^{TS}_{i-1}}{E^L_{i-1}}$		15.27%	15.34%	15.40%	15.46%	15.44%
Levered equity	193.5327	208.9993	224.5690	241.6666	268.8257	
Total levered value	216.6096	239.7686	263.0305	287.8205	314.9796	

Again, the values match.

Now we examine the Adjusted Present value approach to check if keeping the assumptions the values match. We have to realize that the APV when Kd is the discount rate for TS is identical to the present value of the CCF.

Table A9. ψ=Kd: Discount rate Kd: Levered values calculated with APV

Year	2003	2004	2005	2006	2007	2008
FCF with TV_{FCF}		8.2	11.2	12.8	13.8	422.4
TS		1.20	1.60	2.00	2.40	2.40
PV(FCF at Ku)	210.1340	233.6510	257.7178	283.8170	312.8557	
PV(TS at Kd)	6.4757	6.1175	5.3128	4.0034	2.1239	
Total APV	216.6096	239.7686	263.0305	287.8205	314.9796	
Equity	193.5327	208.9993	224.5690	241.6666	268.8257	

Once again the values match because we have used consistent formulations for every method.

Now using the CFE we will calculate the TV for the CFE and levered equity value. When we add the debt balance, we obtain the total levered value. Before calculating the values we have to calculate the terminal value for the CFE, TV_{CFE}, and to do that we need to estimate the growth rate for the CFE, G, or simply subtract from the TV_{FCF} the value of the outstanding debt as we did above. Although it is simpler to subtract the debt from the TV_{FCF}, we will derive the consistent value for G, using equations (19) and (29)

Case a) Considering the debt outstanding at year N and that debt to obtain G, the growth of the CFE (equation (19)).

$$G = \frac{FCF(1+g)K_e - DK_e\left(WACC_{perpetuity} - g\right) - CFE\left(WACC_{perpetuity} - g\right)}{FCF(1+g) - D\left(WACC_{perpetuity} - g\right) + CFE(WACC_{perpetuity} - g)}$$

and calculating the Ke for perpetuity as

$$Ke_{perp} = Ku + (Ku - Kd)(1 - T)\frac{D}{E}$$

As the leverage for perpetuity is 50% the Ke_{perp} is

= 15.094% + (15.094% – 13%)(1 – 40%)50%/50% = 16.35%

The components of equation (19) are shown in the next table in the order they appear in the equation. The last column is the sum of the terms at the left.

Table A10. ψ=Kd: Elements to calculate G, the growth for CFE

	Term 1	Term 2	Term 3	Sum
Numerator	2.589186	−0.3461012	−0.51368333	1.72940146
Denominator	15.836	−2.11682692	0.51368333	14.2328564
			G	12.15%

With this G we calculate the TV_{CFE} as

$$TV_{CFE} = \frac{CFE(1+G)}{Ke_{perp} - G} = \frac{11.20(1+13.46\%)}{16.35\% - 12.15\%} = 299.12$$

From Table 3 we know that the formulation for Ke for finite cash flows and Kd as discount rate for the TS is $Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}} - (Ku_i - Kd_i)\frac{V^{TS}_{i-1}}{E^L_{i-1}}$

Table A11. ψ=Kd: Levered values calculated with CFE and Ke

Year	2003	2004	2005	2006	2007	2008
CFE		14.09	16.49	17.49	10.20	11.20
VT_{CFE}						299,12
CFE + VT_{CFE}		14,09	16,49	17,49	10,20	310,32
Value of TS	6.48	6.12	5.31	4.00	2.12	
Ke		15,27%	15,34%	15,40%	15,46%	15,44%
Levered equity	193,5327	208,9993	224,5690	241,6666	268,8257	
Total levered value	216.6096	239.7686	263.0305	287.8205	314.9796	

Once again, the values match as expected.

At this moment we are not surprised that the values match because in all the methods we have kept the same assumptions and we have used the correct formulations for each set of assumptions. As we can see, all the calculated values, including the calculations for the cost of capital match when we use the consistent assumptions and the proper formulations for each case.

Case b) Considering the level of debt to obtain the perpetual leverage $D\%_{perp}$

In this case we have to consider an additional debt (repayment) to reach the defined perpetual leverage $D\%_{perp.}$ This extra debt will be considered as equity repurchase as will be shown below in Table A13.

Now we have the CFE, the CFD and the TS, hence we can estimate the FCF and the CCF taking into account that the level of debt at year N is $D\%_{perp} \times TV$. But this implies to calculate the terminal value.

Table A12. FCF and CCF derived from CFD, CFE and TS (ψ=Kd)

Year	2003	2004	2005	2006	2007	2008
Debt at beginning	23.08	30.77	38.46	46.15	46.15	46.15
New debt						126.48
Total debt	23.08	30.77	38.46	46.15	46.15	172,64
Interest charges		3.00	4.00	5.00	6.00	6.00
Principal payment		−7.69	−7.69	−7.69	0.00	−126,48
Total CFD		−4.69	−3.69	−2.69	6.00	−120.48
CFE		14.09	16.49	17.49	10.20	11.20
Repurchase of equity						126.48
Total CFE		14.09	16.49	17.49	10.20	137.68
Sum = CFD + CFE		9.40	12.80	14.80	16.20	17.20
TS		1.20	1.60	2.00	2.40	2.40
FCF = CFD + CFE – TS		8.20	11.20	12.80	13.80	14.80
CCF = FCF + TS		9.40	12.80	14.80	16.20	17.20

If we use the expression for G using $D\%_{perp}$ instead of D, the outstanding debt in the forecasting period we apply equation (29).

$$G = \frac{FCF(1+g)(1-D\%_{perp})K_e - CFE(WACC_{perp} - g)}{FCF(1+g)(1-D\%_{perp}) + CFE(WACC_{perp} - g)}$$

We consider that the extra debt (extra payment) is an amount that has to be added (subtracted) to the terminal value for the CFE. The terminal value for the CFE has to be calculated always with the CFE before any equity repurchase because that value is the one generated by the firm to pay to the equity holders.

Applying

$$G = \frac{FCF(1+g)(1-D\%_{perp})K_e - CFE(WACC_{perp} - g)}{FCF(1+g)(1-D\%_{perp}) + CFE(WACC_{perp} - g)}$$

we have

Table A13. Calculation of G assuming ψ=Kd

	Term 1	Term 2	Sum
Numerator	1.29	0.51368333	0.78
Denominator	7.918	0.51368333	8.43168333
		G	9.26%

The calculation for G, the growth for the CFE is made using the expected CFE for the forecasting period not including the "fictitious" repurchase of equity in year N. With this G we can calculate the TV for the CFE assuming ψ = Kd. The terminal value for the CFE is 163.05.

$$TV_{CFE} = \frac{CFE(1+G)}{Ke_{perp} - G} = \frac{11.20(1+9.46\%)}{16.98\% - 9.46\%} = 163.05$$

Then the calculation of the levered equity is

Table A14. Levered values calculated with CFE and Ke using $D\%_{perp}$ and ψ=Kd

	2003	2004	2005	2006	2007	2008
CFE		14.09	16.49	17.49	10.20	11.20
TV_{CFE}						172.64
Repurchase of equity						126.48
Total CFE = CFE + VT_{CFE}		14.09	16.49	17.49	10.20	310.32
TS		1.20	1.60	2.00	2.40	2.40
Value of TS	6.48	6.12	5.31	4.00	2.12	
Ke		15.27%	15.34%	15.40%	15.46%	15.44%
Levered equity	193.5327	208.9993	224.5690	241.6666	268.8257	
Total levered value	216.6096	239.7686	263.0305	287.8205	314.9796	

We can compare with the other tables and find that the levered values and the Ke are identical.

At this moment we are not surprised that the values match because in all the methods we have kept the same assumptions and we have used the correct formulations for each set of assumptions.

Finite cash flows and Ku as the discount rate for the TS

Now we will derive the firm and equity values using several methods and assuming finite cash flows and Ku as the discount rate for the TS.

In order to calculate the terminal value TV, for the FCF we have to estimate the WACC at perpetuity $WACC_{perp}$, given our assumptions. From Table 5 we have

$$WACC_{perp} = Ku_i - TKd\theta = 15.094\% - 40\% \times 13\% \times 50\% = 12.49\%$$

For simplicity and without losing generality, we are assuming that there is no additional investment at perpetuity.

Hence, the terminal value for the FCF, TV_{FCF}, can be calculated as follows

$$TV_{FCF} = \frac{FCF(1+g)}{WACC_{perp} - g} = \frac{14.80(1+7\%)}{12.49\% - 7\%} = 288.25$$

Using the $WACC_{adj}$ from table 5, $Ku_i - \frac{TS_i}{V_{i-1}^L}$ we calculate solving the circularity, the levered value of the firm and the equity. Using the data from Table A4b we calculate the levered values using the FCF and the adjusted WACC.

Table A15. ψ=Ku: Levered values calculated with FCF and Adjusted WACC

Year	2003	2004	2005	2006	2007	2008
FCF		8.20	11.20	12.80	13.80	14.80
TV_{FCF}						288.25
FCF with TV_{FCF}		8.20	11.20	12.80	13.80	303.05
TS		1.20	1.60	2.00	2.40	2.40
$WACC_{adj} = Ku_i - \frac{TS_i}{V_{i-1}^L}$		14.46%	14.32%	14.21%	14.11%	14.19%
Total levered value	188.0174	206.9963	225.4398	244.6671	265.3965	
Levered equity	164.9405	176.2271	186.9782	198.5133	219.2427	

Now using the CCF and the $WACC^{CCF}$ we calculate the same values. From table 6 we know that the $WACC^{CCF}$ is K_u.

Table A16. ψ=Ku: Levered values calculated with CCF and WACC for CCF

Year	2003	2004	2005	2006	2007	2008
CCF		9.40	12.80	14.80	16.20	305.45
$WACC^{CCF}$		15.094%	15.094%	15.094%	15.094%	15.094%
Levered value	188.0174	206.9963	225.4398	244.6671	265.3965	
Levered equity	164.9405	176.2271	186.9782	198.5133	219.2427	

As we expected, the levered values match. It is not strange because we have used the same assumptions and the correct formulations in each case.

Again, as the conditions required using the traditional WACC formulation and the FCF, we present the values calculated using it. From Table 3 we know that the correct formulation for Ke with finite cash flows and Ku as the discount rate for TS is $Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}}$ and the traditional WACC is $Kd_i(1-T)\frac{D_{i-1}}{V^L_{i-1}} + \frac{Ke_i E_{i-1}}{V^L_{i-1}}$.

Table A17. ψ=Ku: Levered values calculated with FCF and traditional WACC

Year	2003	2004	2005	2006	2007	2008
FCF		8.20	11.20	12.80	13.80	14.80
TV^{FCF}						288.25
FCF with TV^{FCF}		8.20	11.20	12.80	13.80	303.05
Kd(1-T)		7.80%	7.80%	7.80%	7.80%	7.80%
KdD%		0.96%	1.16%	1.33%	1.47%	1.36%
Ke = $Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}}$		15.39%	15.46%	15.52%	15.58%	15.53%
KeE%		13.50%	13.16%	12.88%	12.64%	12.83%
Traditional WACC		14.46%	14.32%	14.21%	14.11%	14.19%
Levered value	188.0174	206.9963	225.4398	244.6671	265.3965	
Levered equity	164.9405	176.2271	186.9782	198.5133	219.2427	

As expected, the levered values coincide. Now we calculate the TV for the CFE as the TV for the FCF minus the outstanding debt and the equity value (and total levered value) using the CFE.

Table A18. ψ=Ku: Levered values with the CFE

	2003	2004	2005	2006	2007	2008
CFE		14.09	16.49	17.49	10.20	11.20
$TV_{CFE} = VT_{FCF} - \text{debt}$						242.10
$CFE + VT_{CFE}$		14.0923	16.4923	17.4923	10.2000	253.3010
$Ke\ Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}}$		15.39%	15.46%	15.52%	15.58%	15.53%
Levered equity	164.9405	176.2271	186.9782	198.5133	219.2427	
Total levered value	188.0174	206.9963	225.4398	244.6671	265.3965	

Again, the values match.

Now we examine the Adjusted Present value approach to check if keeping the assumptions the values match. We have to realize that the APV when Ku is the discount rate for TS is identical to the present value of the CCF.

Table A19. ψ=Ku: Levered values calculated with APV

Year	2003	2004	2005	2006	2007	2008
FCF with TV_{FCF}		8.20	11.20	12.80	13.80	303.05
TS		1.20	1.60	2.00	2.40	2.40
PV(FCF at Ku)	181.8990	201.1544	220.3161	240.7701	263.3113	
PV(TS at Ku)	6.1184	5.8419	5.1237	3.8970	2.0853	
Total APV	188.0174	206.9963	225.4398	244.6671	265.3965	
Equity	164.9405	176.2271	186.9782	198.5133	219.2427	

Once again the values match because we have used consistent formulations for every method.

Section Six

Now using the CFE we will calculate the TV for the CFE and levered equity value. When we add the debt balance, we obtain the total levered value. Before calculating the values we have to calculate the terminal value for the CFE, TV_{CFE}, and to do that we need to estimate the growth rate for the CFE, G, or simply subtract from the TV_{FCF} the value of the outstanding debt as we did above. Although it is simpler to subtract the debt from the TV_{FCF}, we will derive the consistent value for G, using equations (19) and (29)

Case a) Considering the debt outstanding at year N and that debt to obtain G, the growth of the CFE (equation (19)).

$$G = \frac{FCF(1+g)K_e - DK_e\left(WACC_{perpetuity} - g\right) - CFE\left(WACC_{perpetuity} - g\right)}{FCF(1+g) - D\left(WACC_{perpetuity} - g\right) + CFE(WACC_{perpetuity} - g)}$$

and calculating the Ke for perpetuity as

$$Ke_{perp} = Ku + (Ku - Kd)\frac{D}{E}$$

As the leverage for perpetuity is 50% the previous formulation reduces to

Ke_{perp} = Ku + (Ku–Kd)D%/E% = 15.094% + (15.094% – 13%)50%/50% = 17.19%

The components of equation (19) are shown in the next table in the order they appear in the equation. The last column is the sum of the terms at the left.

Table A20. ψ=Ku: Elements to calculate G, the growth for CFE

	Term 1	Term 2	Term 3	Sum
Numerator	2.7218125	−0.43580228	−0.6153	1.67071022
Denominator	15.836	−2.53557692	0.6153	13.9157231
			G	12.01%

The calculation for G, the growth for the CFE is made using the expected CFE for the forecasting period not including the “fictitious” repurchase of equity in year N. With this G we calculate the TV_{CFE} as

$$TV_{CFE} = \frac{CFE(1+G)}{Ke_{perp} - G} = \frac{11.20(1+12.01\%)}{17.19\% - 12.01\%} = 242.1010$$

From Table 3 we know that the formulation for Ke for finite cash flows and Kd as discount rate for the TS is $Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E^L_{i-1}}$.

Table A21. ψ=Ku: Levered values calculated with CFE and Ke

Year	2003	2004	2005	2006	2007	2008
CFE		14.09	16.49	17.49	10.20	11.20
VT_{CFE}						242.1010
CFE + TV_{CFE}		14.0923	16.4923	17.4923	10.2000	253.3010
Ke		15.39%	15.46%	15.52%	15.58%	15.53%
Levered equity	164.9405	176.2271	186.9782	198.5133	219.2427	
Levered value	188.0174	206.9963	225.4398	244.6671	265.3965	

Once again, the values match as expected.

As we can see, all the calculated values, including the calculations for the cost of capital match when we use the consistent assumptions and the proper formulations for each case.

Case b) Considering the level of debt to obtain the perpetual leverage $D\%_{perp}$

In this case we have to consider an additional debt (repayment) to reach the defined perpetual leverage $D\%_{perp.}$ This extra debt will be considered as equity repurchase as will be shown below in Table A23.

Now we have the CFE, the CFD and the TS, hence we can estimate the FCF and the CCF taking into account that the level of debt at year N is $D\%_{perp} \times$ TV. But this implies to calculate the terminal value.

Table A22. ψ=Ku: FCF and CCF derived from CFD, CFE and TS

Year	2003	2004	2005	2006	2007	2008
Debt at beginning	23.08	30.77	38.46	46.15	46.15	46.15
New debt						97.97
Total debt	23.08	30.77	38.46	46.15	46.15	144.13
Interest charges		3.000	4.000	5.000	6.000	6.000
Principal payment		−7.69	−7.69	−7.69	0.00	−97.97
Total CFD		−4.69	−3.69	−2.69	6.00	−91.97
CFE		14.09	16.49	17.49	10.20	11.20
Repurchase of equity						97.97
Total CFE		14.09	16.49	17.49	10.20	109.17
Sum = CFD + CFE		9.40	12.80	14.80	16.20	17.20
TS		1.20	1.60	2.00	2.40	2.40
FCF = CFD + CFE – TS		8.20	11.20	12.80	13.80	14.80
CCF = FCF + TS		9.40	12.80	14.80	16.20	17.20

If we use the expression for G using $D\%_{perp}$ instead of D, the outstanding debt in the forecasting period we apply equation (29).

$$G = \frac{FCF(1+g)(1-D\%_{perp})K_e - CFE(WACC_{perp} - g)}{FCF(1+g)(1-D\%_{perp}) + CFE(WACC_{perp} - g)}$$

We consider that the extra debt (extra payment) is an amount that has to be added (subtracted) to the terminal value for the CFE. The terminal value for the CFE has to be calculated always with the CFE before any equity repurchase because that value is the one generated by the firm to pay to the equity holders.

Applying eq (29) we have

Table A23. Calculation of G assuming ψ=Ku

	Term 1	Term 2	Sum
Numerator	1.36090625	-0.6153	0.74560625
Denominator	7.918	0.6153	8.5333
G			8.74%

The calculation for G, the growth for the CFE is made using the expected CFE for the forecasting period not including the "fictitious" repurchase of equity in year N. With this G we can calculate the TV for the CFE assuming ψ = Ku. The terminal value is

$$TV_{CFE} = \frac{CFE(1+G)}{Ke_{perp} - G} = \frac{11.20(1+8.74\%)}{17.19\% - 8.74\%} = 144.13$$

Then the calculation of the levered equity is

Table A24. Calculation of levered equity using $D\%_{perp}$ and ψ=Ku

	2003	2004	2005	2006	2007	2008
CFE		14.09	16.49	17.49	10.20	11.20
TV CFE						144.13
Repurchase of equity						97.97
Total CFE		14.0923	16.4923	17.4923	10.2000	253.3010
Ke		15.39%	15.46%	15.52%	15.58%	15.53%
Levered equity	164.9405	176.2271	186.9782	198.5133	219.2427	
Total levered value	188.0174	206.9963	225.4398	244.6671	265.3965	

We can compare with table A15 and find that the levered values and the Ke are identical.

With this example we have shown that when done properly, we can arrive to the correct levered values using the FCF, the CCF or the CFE. When using the CFE we can derive its terminal value subtracting outstanding debt from the TV for the FCF or calculating the terminal value for the CFE either taking into account the outstanding debt at year N or taking into account the total debt as defined by the perpetual leverage, $D\%_{perp}$.

From the exploration of the calculated levered values we observe that the values obtained when we assume Ku as the discount rate are higher than those calculated with Kd as the discount rate. This is not a surprise because the former does not have the (1 – T) factor in the calculation of the Ke and this makes the discount rates higher than when we use the (1 – T) factor in the calculation of Ke. The differences between levered values are as shown in the next table.

Table A25. Differences in using Kd or Ku as assumption regarding the discount rate for TS

Discount rate for TS	Kd	Ku	Difference
Total levered value	216.61	188.02	15.21%
Levered equity	193.53	164.94	17.33%

In pointing out these differences we are not claiming that one assumption is correct and the other is incorrect. That is a debate that has not concluded.